ChPT calculations of pion formfactors

Karol Kampf[a]

[a] *Institute of Particle and Nuclear Physics, Faculty of Mathematics and Physics, Charles University, V Holešovičkách 2, 18000 Prague, Czech Republic*

Abstract

An overview on chiral perturbation theory calculations of form factors is presented. The main focus is given on the form factors related to the lightest meson, pion, namely: pion decay constant, pion vector and scalar form factor, radiative pion decay and transition form factor. A pure calculation within the effective theory can be extended using further methods, as resonance chiral theory and leading logarithm calculations.

Keywords: Chiral Lagrangians, 1/N Expansion, radiative decay of π^0

1. Introduction

The formfactors of quantum chromodynamics (QCD) are well defined objects which can be studied both theoretically and experimentally. We will focus on several basic quantities which are connected with π meson and summarize basic status of their theoretical calculations mainly at low energies, i.e. at the domain of chiral perturbation theory (ChPT). The formfactors connected for example with kaons will not be considered here, but one should note that they play also important role in connection with ChPT (e.g. $K_{\ell 4}$).

2. Pion decay constant

The most simplest formfactor, pion decay constant, is defined in QCD via the coupling of axial current and pion as

$$\langle 0|A^a_\mu(x)|\pi^b(p)\rangle = i\delta^{ab}F_\pi p_\mu \mathrm{e}^{-ipx}\,. \tag{1}$$

As the pion is real, $p^2 = m_\pi^2$, the momentum dependence is trivial and F_π is a constant. This is a reason why it is usually not referred as formfactor in the literature (on recent review see e.g. [1] and references therein). It is a fundamental order parameter of the spontaneous symmetry breaking of $SU(N_f)_L \times SU(N_f)_R$ to $SU(N_f)_V$ of QCD (N_f represents number of light quark flavours, 2 or 3 for real QCD). Its value can be set from the $\pi_{\ell 2}$ decay using Marciano and Sirlin formula for radiative corrections [2]. Updated by virtual photons [3] and V_{ud} value [4] one can obtain [5]

$$F_\pi = 92.215 \pm 0.0625\,\mathrm{MeV}\,. \tag{2}$$

In pure QCD, F_{π^0} and $F_{\pi^\pm}$ difference is NNLO effect and this was evaluated in [5] and found to be indeed very small. We use this fact in order to set the pion decay constant form π^0 lifetime. Using the NNLO calculation within ChPT of $\pi^0 \to \gamma\gamma$ decay [5] subtracting QED corrections one can arrive to

$$F_{\pi^0} = 93.85 \pm 1.3\,(\text{exp.}) \;\pm 0.6\,(\text{theory})\,\mathrm{MeV}\,. \tag{3}$$

As an experimental input the PrimEx measurement was used [6]. We can see that the precision obtained here cannot still compete with the precision obtained using charged pion decay. However new experimental activity (e.g. PrimEx2, KLOE-II) can improve the experimental error. On the theory side there are also possible improvements foreseen. One of them, the full calculation of the $\eta \to \gamma\gamma$ decay will be valuable [7], as well as a better estimation of the value of the isospin breaking coefficient $\sim (m_d - m_u)$. What is important to stress at this moment is that possible tension between these two values (2) and (3) can be attributed to new physics: F_π, determined from the weak decay of the π^+ assumed the validity of the standard model. A possible deviation from it via right-handed currents was opened in [8].

Email address: karol.kampf@mff.cuni.cz (Karol Kampf)

3. Electromagnetic formfactor of charged pion

Vector or more precisely electromagnetic formfactor of the charged pion, F_V^π is defined by

$$\langle \pi^+(p_f)|j_\mu^{\mathrm{elm}}|\pi^+(p_i)\rangle = (p_f + p_i)_\mu F_V^\pi[(p_f - p_i)^2]\,. \quad (4)$$

Its calculation within ChPT up to two-loop level can be found in [9] and using dispersive treatment in [10]. Data within the validity of ChPT were taken so far mainly from a space like region (cf.[9]). New measurements in a time-like region almost down to the di-pion threshold (at KLOE10 [11]) urge us to answer the question of validity of ChPT more precisely. For this we will turn to the calculations of the leading logarithms.

Leading logarithms (LL) are logarithms with highest possible power at the given order. Similarly as in the renormalizable theory they can be calculated using only one-loop diagrams [12]. In the renormalizable theory their summation has an important phenomenological consequence: the running coupling constant. In effective theory, as ChPT, the LL coefficients are given only by the form of the leading-order Lagrangian. They are thus parameter-free and without further knowledge of low-energy constants can be used as a rough estimate of the given order. However, the general method for their summation is not known and thus at the moment we must rely only on some simplify cases where it was possible. LL were calculated up to the fifth order in the massive $O(N)$ model (for $N = 3$ it is equivalent to two-flavour ChPT) in [13]. In the massless and large N limit it is indeed possible to resum all LL, the closed form is (cf. also [14], but mind the sign)

$$F_V^{0NLN}(t) = 1+\frac{1}{N}+\frac{4}{K_tN^2}\Big[1-\Big(1+\frac{2}{K_tN}\Big)\log\Big(1+\frac{K_tN}{2}\Big)\Big], \quad (5)$$

with

$$K_t \equiv \frac{t}{16\pi^2F^2}\log\Big(-\frac{\mu^2}{t}\Big). \quad (6)$$

The calculated LL together with the resummed function is depicted in Fig. 1. It is clear that convergence in the time-like region is already problematic not far above the threshold. It also shows how important is a resum function (at least in the studied limits). Let us note that even LL are very important in studying the convergence, the actual numerical value is still dominated by the large higher-order coefficients [9].

4. Scalar formfactor

The definition reads

$$F_S^\pi(t \equiv (p-q)^2) = \langle\pi^0(q)|\bar{u}u + \bar{d}d|\pi^0(p)\rangle \quad (7)$$

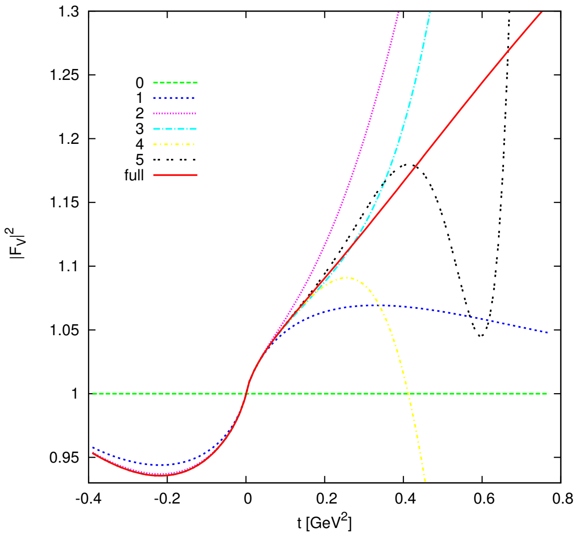

Figure 1: The leading-logarithms for normalized F_V in massless and large N limit in $O(N = 3)$ model.

Its calculation within ChPT exists up to next-to-next-to-leading order in [15]. The fact that this quantity cannot be practically measured today can appear as a problem. However, using $\pi\pi$ phase shifts and the dispersive treatment [16] we can study its energy dependence. Writing

$$F_S^\pi(t) = F_S^\pi(0)\Big(1+\frac{1}{6}\langle r^2\rangle_S^\pi t + c_S^\pi t^2 + \ldots\Big), \quad (8)$$

one would obtain

$$\langle r^2\rangle_S^\pi = 0.61 \pm 0.04 \text{ fm}^2, \qquad c_S^\pi = 11 \pm 2 \text{ GeV}^{-4}. \quad (9)$$

These values were recently used in the new global fit of low energy constants of the 3-flavour ChPT [17].

5. Radiative pion decay

The pion decay $\pi^+ \to e^+\nu\gamma$ (see e.g. works in [18]) is interesting in the context of the QCD formfactors because its structure dependent part dominates over the inner Bremsstrahlung due to the helicity suppression. The structure dependent part, connected to the pionic structure, can be further decomposed to the vectorial ($\sim F_V$) and axial ($\sim F_A$) part. Beyond standard model one can consider also tensor radiation part ($\sim F_T$). As there is no significant hint from the recent measurements – the most precise limits are $F_T = (-0.6 \pm 2.8) \times 10^{-4}$ set by the PIBETA group [19] – we will not consider it here. The vector part of the $V - A$ structure, defined as ($e = 1$ for simplicity)

$$\int d^4x e^{iq.x}\langle 0|T(j_\mu^{\mathrm{elm}}(x)j_\nu^{V;1-i2}(0)|\pi^+(p)\rangle = \epsilon_{\mu\nu\alpha\beta}q^\alpha p^\beta \frac{F_V}{m_{\pi^+}}$$

and $\pi^0\gamma\gamma$ amplitude, defined as

$$\int d^4x e^{iq.x}\langle 0|T(j^{\mathrm{elm}}_\mu(x) j^{\mathrm{elm}}_\nu(0)|\pi^+(p)\rangle = \epsilon_{\mu\nu\alpha\beta}q^\alpha p^\beta A_{\pi\gamma\gamma} \tag{10}$$

can be connected employing an isospin symmetry

$$\sqrt{2}\frac{F_V}{m_{\pi^+}} = A_{\pi^0\gamma\gamma}\,. \tag{11}$$

The recent measurement [19] $F_V = 0.0258(17)$ is in agreement with the value obtained either from $\pi^0 \to \gamma\gamma$ decay width or $O(p^4)$ theoretical calculation. This value was also used as an independent determination of the neutral pion lifetime (see also [4])

$$\tau^{\mathrm{PSI}}_{\pi^0} = (8.5 \pm 1.1) \times 10^{-17}\ \mathrm{s}\,. \tag{12}$$

However, one should be careful with systematic uncertainties. As we have mentioned the connection between F_V and $\pi^0\gamma\gamma$ is based on the isospin symmetry. It also means that the value of the mass of pion in (11) is just matter of convention. The dependence on the actual value is source of roughly 50% of the error in (12). Independently of $\pi_{l\gamma}$ decay, the isospin-breaking corrections were found to be very important also in the theoretical estimate of $\pi^0 \to \gamma\gamma$ decay width [5].

6. Transition formfactor

The transition pion-gamma-gamma (all off-shell) formfactor is a quantity accessible via a definition of the QCD Green function of the vector-vector and pseudoscalar currents

$$\Pi^{abc}_{\mu\nu}(p,q) = \int dxdy e^{ip.x+iq.y}\langle 0|T[V^a_\mu(x)V^b_\nu(y)P^c(0)]|0\rangle\,. \tag{13}$$

Using Ward identities and Lorentz and parity invariance we can extract

$$\Pi^{abc}_{\mu\nu}(p,q) = d^{a,b,c}\epsilon_{\mu\nu\alpha\beta}p^\alpha q^\beta \Pi(p^2,q^2;r^2 \equiv (p+q)^2)\,.$$

The formfactors can be obtained using the LSZ. E.g. for the π^0 transition formfactor we have (for simplicity in the chiral limit)

$$\mathcal{F}_{\pi^0\gamma\gamma}(p^2,q^2;r^2) = \frac{2}{3}\frac{1}{BF}r^2\Pi(p^2,q^2;r^2)\,. \tag{14}$$

The application of this object is very wide. The most important place where its theoretical behaviour is most desired is probably the hadronic light-by-light contribution in muonic anomalous magnetic moment. Putting a pion on shell we can consider two regions depending on a photon virtuality: space-like (represented e.g. by the e^+e^- fusion to π^0) and time-like region (e.g. $\pi^0 \to e^+e^-\gamma$). On the more detailed overview and literature see the recent MesonNet workshop [20].

The area of the applicability of ChPT is demonstrated on Fig. 2. We clearly see that area is roughly

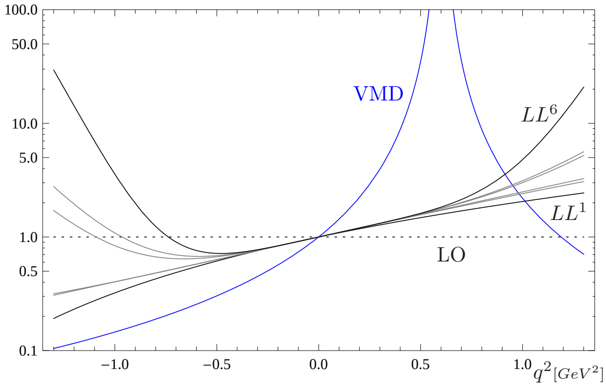

Figure 2: Normalized $|\mathcal{F}_{\pi\gamma\gamma^*}|^2$ as a function of a virtual photon, calculated using LL only (up to 6-loop order) [e.g. $LO = 1$, $LL^1 = 1 + \frac{q^2}{96\pi^2 F_\pi^2}\ln\frac{\mu^2}{m_\pi^2}$]. For comparison: vector meson dominance curve (VMD).

$|q^2| \lesssim 0.5$ GeV for both time-like and space-like q. Unfortunately this region is not yet very well covered by experimental data. In the following we will discuss both regions in more detail.

6.1. *Space-like region*

Apart from the model-independent LL we must employ some specific model and parameters in order to describe behaviour of the objects as defined for example in (14) at low energies. In pure ChPT we have to deal with low-energy constants. For $\mathcal{F}_{\pi^0\gamma\gamma}$ these are mainly C^W_7 and C^W_{22} of the odd-intrinsic-parity sector [21]. They must incorporate the existence of resonances and their effect even below their thresholds (as it is clear in Fig. 2). We will be, however, still limited with the applicability of such models strictly below these resonances. On the other hand, one can enlarge ChPT by resonances and keep them as active degrees of freedom. For the mentioned odd sector this was studied systematically in [22]. Generally, the base or Lagrangian of the lowest lying resonances for VVP gives complicated result with many parameters. Using the operator-product expansion they are reduced just to two parameters. This verifies the so-called LMD+P ansatz [23]. For the on-shell pion we have only one parameter left and this can be set using transition form factor. Another way how to set this parameter is to use the information on $\rho \to \pi\gamma$ decay. However, the experimental error of this value is still big. Nevertheless one can use it as a consistency

check and it seems in good agreement [22]. The second parameter, which is connected with the off-shell pion can be obtained from information on $\pi(1300) \to \rho\gamma$ and $\pi(1300) \to \gamma\gamma$. However, here the experimental situation is even worse. Fortunately, there is at least one experimental information from Belle: limit on $\pi(1300) \to \gamma\gamma$ [24].

To summarize using the phenomenological information (from the space-like region) we may set all relevant parameters within resonance chiral theory and make some non-trivial predictions.

6.2. Time-like region

In the time-like region the transition formfactor $\mathcal{F}_{\pi^0\gamma\gamma}$ is mainly connected with the Dalitz decay $\pi^0 \to e^+e^-\gamma$ (see recent [25] and references therein). For the needs of the low-energy region it is convenient to study only a slope parameter a_π:

$$\mathcal{F}_{\pi^0\gamma\gamma^*}(q^2) = \mathcal{F}_{\pi^0\gamma\gamma^*}(0)\Big(1 + a_\pi \frac{q^2}{m_{\pi^0}^2} + \dots\Big) . \tag{15}$$

Having the experimental data one should extract first the QED corrections:

$$\frac{\mathrm{d}\Gamma^{exp}}{\mathrm{d}x} - \delta_{QED}(x)\,\frac{\mathrm{d}\Gamma^{LO}}{\mathrm{d}x} = \frac{\mathrm{d}\Gamma^{LO}}{\mathrm{d}x}\,[1 + 2x\,a_\pi] . \tag{16}$$

These QED corrections are well understood [25] and they include now also one-photon irreducible contributions. Its value

$$\delta a_\pi \Big|_{1\gamma IR} \doteq 0.005 \tag{17}$$

should be subtracted from the two relevant experiments [26]. The central value would shift to the excellent agreement with the theoretical prediction

$$a_\pi^{\mathrm{theo}} = 0.029 \pm 0.005 . \tag{18}$$

However, one should note that huge experimental errors (more than 100%) make the comparison meaningless.

7. Conclusions

We have studied some basic properties of the pion formfactors at low energy. We have first briefly discussed pion decay constant F_π and set possible inconsistency in this value obtained using $\pi_{\ell 2}$ and $\pi^0 \to \gamma\gamma$. On the next object, electromagnetic formfactor of charged pion we have demonstrated use of the so-called leading logarithms in studying the convergence. Short overviews on scalar formfactor and radiative pion decay were also given. Last but not least, $\pi - \gamma - \gamma$ transition formfactor was discussed both for time-like and space-like region.

Acknowledgements

The work is supported by projects MSM0021620859 of Ministry of Education of the Czech Republic.

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